

QUESTIONS OF THE DIAGNOSTICS OF HIGH-TEMPERATURE
MATERIAL TESTING REGIMES BY USING INVERSE
PROBLEMS

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A comparative analysis is performed of the efficiency and operability of heat-flux sensors in checking out the temperature regimes of textile material tests. Questions of the practical application of inverse heat-conduction problem (IHCP) sensors are considered.

The practice of characterizing the thermal specimen loading conditions during experiments performed on the temperature in different zones over the material thickness as well as on its surface has traditionally been complicated in tests.

Mounting thermocouples with less than $0.1 \cdot 10^{-3}$ m electrode thickness directly in the specimen is irrational in mass testings of textile materials since it is fraught with great technical difficulties. A high heat-flux level to the material is noted in tests in short pulse regimes, and correspondingly, so is a substantial nonstationarity of the temperature field with significant temperature drops over the specimen thickness.

Available literature data on heat conduction and methods of measuring the temperature field of fabric structures, for instance [1-3], permit making the deduction that measuring the temperature on the surface and at different points in the bulk of the fabric or tape by using even quite thin thermocouples cannot often yield a unique answer. Hence, in a number of experiments with textile materials under unilateral heating, a transition is made to the identification of the thermal test conditions by the time dependence of the heat flux incident on the specimen.

To program the values of the heat flux it is necessary to have radiant flux sensors (RFS) that are reliable in construction and correct in operating principle. It is desirable that these be direct reading sensors.

For short heating cycles on the order of 0.5-1.0 sec, all the known direct measurement RFS turn out to be extremely inertial. On the basis of the data in [3-6], the deduction can be made that the most low-inertia RFS of the Gordon type [6] probably have a time constant on the order of 0.3 sec and a time of arrival at a constant reading of 0.7 sec for a rectangular heat flux pulse at this time.

Analytic investigations of the inertia of membrane RFS (Gordon, TsAGI type) also show that they have insuperable disadvantages, the lag of the readings in the rapid processes characterizing the high rate of change of the incoming radiant energy flux q_r . Different methods of solving the energy equations for the RFS sensor yield an approximately identical estimate of the inertia. Thus, the estimate of the time of the transient in the rectangular pulse $\tau_{tr} = (0.65-1.05) R^2/a$ is obtained in [4, 5], where a is the thermal diffusivity of membrane material of radius R .

If q_r changes by a linear law, the heat flux sensor reading (the temperature at the center of the sensor membrane) will have a constant error proportional to τ_{tr} and the Kirpichev criterion Ki . The relative contribution of the error will diminish with heating time.

Therefore, for thermal action times on the order of 1 sec with a definite nonstationarity of the heat flux, the application of the most perfect direct-heating RFS can yield a substantial inaccuracy in the estimation of the heat supply and the reproduction of given heating

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conditions. The necessity occurs to produce methods of correcting the RFS readings in the initial section (in the transients) of the thermal action. However, as an analysis of the literature on thermal experiments shows, such methods cannot, as a rule, be realized by using direct reading facilities and require subsequent mathematical processing of the primary data. Methods of solving inverse heat-conduction problems (IHCP) that permit, in particular, the restoration of thermal boundary conditions on the heat perceptive surface of the body [7-9] comprise the theoretical basis for mathematical support of thermal experiments of the type under consideration. In application to RFS of different kinds, when solving IHCP on the basis of recorded data on the nonstationary temperature field of the transducer sensor, boundary conditions (BC) are sought on its surface — the heat flux density q and the surface temperature T_w .

In the general formulation, inverse heat-conduction problems are incorrect in the sense of the absence of a continuous dependence of the results of the solution on the input data, which is especially essential for applied problems when the input information (the RFS readings) contains measurement and decoding errors. Consequently, in order not to allow a rise in the error level during the electronic computer computation, the IHCP solution should be based on such approximate methods as can suppress the instability of the results while simultaneously conserving the accuracy required.

One of the possible methods of constructing a stable algorithm for the solution of IHCP is based on the principle of step regularization and is related to the determination of the least allowable step in the time calculation. However, practical application of such an approach is often unacceptable for the domain under consideration since the magnitude of the critical step can turn out to be longer than the period of emitter and sensor nonstationarity.

Effective algorithms for the solution of the IHCP, which have no constraints on the computational step, can be constructed by using regularization [8] and iterative regularization [9, 10] methods.

Formulation of the IHCP was realized in this paper for the construction of special heat-flux sensors. In experimental investigations conducted in heating by using a KIM-1000 lamp, the operability and efficiency of sensors constructed on the IHCP principle were analyzed as compared with known direct-reading Gordon and TsAGI-type sensor systems.

The inverse heat-conduction problem was solved for the model of an "infinite" plate of thickness b , whose inner surface was considered heat-insulated, and the thermophysical material characteristics were considered temperature-independent. A temperature change was recorded over the thickness b at a certain point x_1 at a distance from the heat-perceptive surface of the plate during the testing.

The mathematical formulation of the problem had the following form

$$\frac{\partial T(x, \tau)}{\partial \tau} = a \frac{\partial^2 T(x, \tau)}{\partial x^2}, \quad \tau \in (0, \tau_m], x \in (0, b]; \quad (1)$$

$$T(x, 0) = T_0; \quad T(x_1, \tau) = T_{\text{mea}}(\tau); \quad (2)$$

$$-\lambda \frac{\partial T(b, \tau)}{\partial x} = q_{\text{in}} = 0; \quad (3)$$

$$-\lambda \frac{\partial T(0, \tau)}{\partial x} = q(\tau) - ? \quad (4)$$

In the formulation under consideration, it reduces to the solution of the integral equation

$$Aq \equiv \int_0^{\tau_m} K(x, \tau - \xi) d\xi = \Theta(x_1, \tau) - \Theta_0 = \Theta_1, \quad (5)$$

where $q(\tau)$ is the desired function of the heat flux, $\Theta(\tau)$ is the known function of the model temperature, $\Theta = \frac{1}{\lambda_0} \int_0^{\tau} \lambda(T) dT$, which permits taking account of the change in the thermophysical

properties $\lambda(T)$, $C(T)$ within the limits of a linear formulation of the problem, and $K(x, \tau)$ is the kernel of the integral equation determined by the physical model under consideration for the heat conduction.

The solution of the problem is based on the regularization method [8], in conformity with which the desired heat flux q is determined from minimization of a functional of the form

$$\Phi[\alpha, q] = \int_0^{\tau_m} [Aq(\tau) - \Theta_1(\tau)]^2 d\tau + \alpha \int_0^{\tau_m} \left[\frac{dq}{d\tau} \right]^2 d\tau. \quad (6)$$

For a step approximation, (6) is written as follows:

$$\Phi_{\Delta\tau}[\alpha, q] = \sum_{n=1}^m \left\{ \sum_{i=1}^n \varphi_{in} \bar{q}_i - \bar{\Theta}_{1n} \right\}^2 \Delta\tau + \alpha \sum_{i=1}^m \frac{(\bar{q}_{i+1} - \bar{q}_i)^2}{\Delta\tau}. \quad (7)$$

Minimization of (7) with respect to q reduces the problem under consideration to the solution of a system of linear algebraic equations

$$\sum_{l=1}^m a_{lk} \bar{q}_l = \bar{f}_k, \quad k = 1, 2, \dots, m; \quad (8)$$

$$a_{lk} = \Delta\tau^2 \sum_{n=l}^m \varphi_{kn} \varphi_{ln} - \alpha, \quad l = k + 1, l = k - 1;$$

$$a_{lk} = \Delta\tau^2 \sum_{n=l}^m \varphi_{kn} \varphi_{ln}, \quad l \geq k + 2, l \leq k - 2;$$

$$a_{ll} = \Delta\tau^2 \sum_{n=l}^m (\varphi_{ln})^2 + 2\alpha, \quad l \neq 1, m;$$

$$a_{ll} = \Delta\tau^2 \sum_{n=l}^m (\varphi_{ln})^2 + \alpha, \quad l = 1, m;$$

$$\bar{f}_1 = \sum_{n=1}^m b_{1n} \Theta_{1n} - \alpha C_1 \Delta\tau;$$

$$\bar{f}_m = b_{mm} \Theta_{1m} + \alpha C_2 \Delta\tau;$$

$$\bar{f}_k = \sum_{n=k}^m b_{kn} \Theta_{1n}, \quad k \neq 1, m;$$

$$l = \Delta\tau^{3/2} \varphi_{kn}; C_1 = \frac{q_1 - q_0}{\Delta\tau}; C_2 = \frac{q_m - q_{m-1}}{\Delta\tau}.$$

The coefficients φ_{kn} are determined by the following expressions [9]:

for $\Delta Fo = \alpha \Delta\tau / b^2 > 0.4$

$$\varphi_{kn} = 1 - \frac{2}{\Delta Fo} \sum_{i=1}^{N_2} \frac{(-1)^{i+1}}{\mu_i^2} \cos(\mu_i \xi) \{ \exp[-\mu_i^2 \Delta Fo(n-k)] - \exp[-\mu_i^2 \Delta Fo(n-k+1)] \},$$

$$\mu_i = i\pi; \xi = 1 - \frac{x_1}{b}; \pi = 3.14159265;$$

for $\Delta Fo \leq 0.4$

$$\varphi_{kn} = -2 \sqrt{(n-l)\Delta Fo} \sum_{j=1}^{N_2} \left\{ i\Phi^* \left[\frac{2j + \frac{x_1}{b}}{2\sqrt{\Delta Fo(n-l)}} \right] + i\Phi^* \left[\frac{2(j+1) - \frac{x_1}{b}}{2\sqrt{\Delta Fo(n-l)}} \right] \right\}_{l=k-1}^{l=k}$$

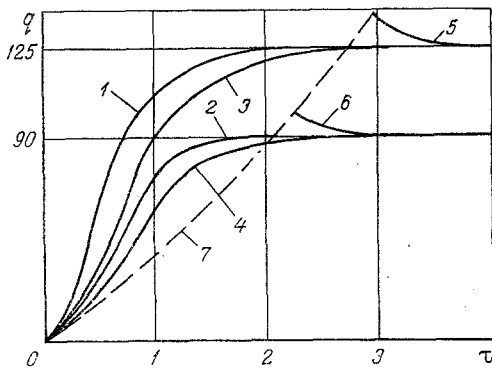


Fig. 1

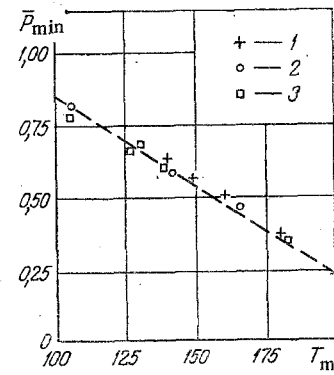


Fig. 2

Fig. 1. Comparison of the results of diagnosing the heat-flux density by different methods: 1, 2) values obtained from solving the IHCP; 3, 4) data of Gordon-type sensor readings; 5, 6) TsAGI radiant flux sensor readings for given values of the heat-flux density 125 (1, 3, 5) and 90 kW/m² (2, 4, 6); 7) provisionally denoted interval of TsAGI sensor inertia.

Fig. 2. Data of an experimental determination of the diminution in the strength of fabric materials \bar{P}_{\min} from the mean temperature over the specimen thickness T_m : 1) capron fabric; 2) tapes for modeling the thermal load "by thermocouple"; 3) tapes for a programmed change in the heat flux.

The summation limits N_1 and N_2 are determined from the accurately given condition on the smallness of the last term of the series.

The system of equations (8) is solved by the square-root method for a value α of the regularization parameter that lies in the selected range of its variation corresponding to the error level in the temperature measurements.

The experimental results obtained permitted execution of a practical comparison between inertia of different methods and the determination of the degree of approach of the heat flux values to the actual values in the heating transients. The thermal inertia of heat flux sensors of the Gordon system with $2 \cdot 10^{-3}$ m diaphragm diameter is 0.7-1.0 sec according to the data in [4-6, 11], while it is approximately 2-3 sec for the TsAGI sensor systems with two thermal resistors of area $(5 \times 5) \cdot 10^{-6}$ m², which are recommended for stationary heating conditions.

An oscillogram of the transient in the investigations performed was made by using the Gordon sensor and the heat-flux sensor constructed by the method of the inverse problems. The results of the investigations are represented in Fig. 1.

The steady values of the heat flux restored from the IHCP solution are practically in agreement with the Gordon and TsAGI sensor readings, which indicates the operability and confidence of the diagnosis methods using the principle of the inverse problems.

The most important result of the experimental investigations performed is the determination of the initial transient section of the heating. A discrepancy in the sensor readings in the initial section is observed for the existing agreement between the heat flux values in the stationary section at the 90- and 125-kW/m² levels. Because of its high inertia (dashed line in Fig. 1), the TsAGI-type sensor does not generally determine the process being observed. Because of the thermal inertia of the membrane resulting in a lag in the readings from the actual process, the Gordon sensor falsely determines the transient section. The values of the heat flux are here lowered, and are 0.84 and 0.9 of the real values for $\tau = 0.55$ sec. The initial section for the change in the heat flux, obtained from the solution of the IHCP, is not subject to the effects of inertia and corresponds to the true values of the flux since the temperature measured in the heat flux sensor being used contains complete information about the external heat transfer.

Results of diagnosing the thermal loading of the specimens permitted a detailed calibration of the heating times on a radiant heating apparatus. The information obtained permitted successful testing of the thermal rupture strength of capron textile tape and fabric under a "thermal impact" simulated by unilateral nonstationary heating by using quartz infrared lamps. The maximum values of the heat flux were 200 kW/m^2 with arrival at the maximum in 0.7 sec and subsequent exponential decay in 2 sec. The rupture strength is characterized by the minimal values and their changes reflect the pattern of the thermal action.

The correlation between relative value of the strength characteristics $\bar{p}_{\min} = P_{\min}(T)/P_{\text{ini}}$ and the mean temperature T_m of the capron textile specimen calculated for nonstationary heat-conduction conditions [1, 2] was clarified in an extension of the experimental results simulating fabric exploitation conditions. The dependence $\bar{p}_{\min} \approx 1.43 - 0.0058T_m$ is obtained for the temperature range $100 < T_m < 200^\circ$ (Fig. 2) for the textile materials investigated, which were of simple structure without additional warp and sewing threads, etc.

The advantage of using heat-flux sensors constructed on the principles of the inverse problems over Gordon and TsAGI system sensors to identify the process of thermal loading in short pulse regimes is verified as a result of the investigations performed. By using the sensors mentioned, a number of experimental results are obtained about the thermal strength of fabrics which are of great scientific and practical value.

NOTATION

τ , time; q_r , incident radiant heat-flux density; α , λ , thermal diffusivity and conductivity coefficients; R , membrane radius; T , temperature; α , regularization parameter; Fo , Fourier number; b , thickness; x_1 , coordinate; Θ , model temperature; \bar{p} , strength attenuation coefficient. Subscripts: min, minimal; m, mean; in, inner; mea, measured; ini, initial.

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